A Recent Controversy on Marxian Fundamental Theorem in Japan

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1. Introduction

I introduce in this paper an outline of a recent controversy in Japan, which is carried out over Marxian Fundamental Theorem.

It is performed between Professor Yoshihara in Hitotsubashi University and me. In Yoshihara (2001), Professor Yoshihara criticized Marxian Fundamental Theorem (MFT) on some points. I refuted two points of them in Matsuo (2004). One is Professor Yoshihara's argument that there can exist some individual workers who are not exploited under positive profit if we allow the different consumption bundles among the working class. The other is the so-called The Generalized Commodity Exploitation Theorem (GCET). On the former argument, I criticized Professor Yoshihara's numerical example. On the latter argument, I criticized GCET regarding the concept of the labor value system. In this context, I used my original concept of exploitation, which had introduced for another problem about MFT before.

Then Professor Yoshihara re-criticized me in Yoshihara (2005). On the former point, he recognized my criticism and showed another numerical example of the existence of the individual workers not exploited under positive profit, and showed the general condition of it. On the latter point, he criticized my new concept of exploitation, besides the counter arguments against my interpretation about the labor value system.

Refuting this, I wrote Matsuo (2006) that is currently under review for journal publication. The arguments in this will be also introduced here.

2. Proposition of the Existence of Unexploited Individual

Yoshihara (2001) said that there can exist some individual workers who are not exploited under positive profit if we allow the different consumption bundles among the working class, although the bestowed labor value of the average real wage bundle for unit labor is less than unity. As Matsuo (2004) showed and Yoshihara (2005) admitted, Yoshihara's (2001) numerical example shown to prove this was not correct, but Yoshihara's (2005) new example and general demonstration are correct. Here I show the simplest example to show the spirit of Yoshihara's (2005) argument.

Now, let there be one joint production process, which yields goods-1 and goods-2. From the input of unit labor, the net product of goods-1 is one and net product of goods-2 is also one, which is shown as the point \mathbf{y} of Figure 1. And let the worker's budget constraint line from the supply of the unit labor be the black downward slop line, where relative price evaluates one goods-1 as one goods-2.

Let there be two types of workers. One type of them prefers only goods-1, whereas the other type of them prefers only goods-2. Therefore, a type-1 worker gets



1.5 units of goods-1, which is shown as the intercept of the horizontal axis of the Figure 1. A type-2 worker gets 1.5 units of goods-2, which is shown as the intercept of the vertical axis of the Figure 1. When these two types of workers employed as half and half, average real wage bundle of the whole economy **R** is shown as the midpoint of the budget constraint line (0.75, 0.75).

Apparently, as the net products are (1, 1) and from which workers get (0.75, 0.75), surplus products of (0.25, 0.25) remain in the hands of the capitalist class. This means there exists positive profit.

But see each worker's real wage bundle. Providing one unit of labor, type-1 worker gets 1.5 units of goods-1, which require 1.5 units of labor for the net production, and type-2 worker gets 1.5 units of goods-2, which require 1.5 units of labor for the net production. Both types of them get products of more amount of labor than they worked. This means no worker is exploited individually but there exists positive profit.

This situation stands under general net production frontier per unit labor, which can be shown as a downward curve projecting out to northeast. If the both intercepts of the budget constraint line lie outside of the frontier, and if the line passes through the inside of the frontier, the example above just stands as it is.

3. Exploitation defined by Minimum Labor with Equal Utility

One of the Yoshihara's (2005) criticisms against my argument is about my original concept of exploitation, which is defined by minimum labor with equal utility. This exploitation concept had been introduced to solve the other problem, which was pointed out by Roemer (1980) and Petri (1980) in the context of criticism against FMT. I will show this criticism based on Petri's (1980) spirit by Figure 2 below.

Let there be a single process and that this process produces two types of goods.



Net products per unit labor input are presented by the point $\mathbf{y} = (y_1, y_2)$, where y_i denotes net products of i-th goods per unit of labor input. And the real wage basket for unit labor is represented by the point **R**. Then as shown in Figure 2, $\mathbf{R} = (y_1, R_2), R_2 < y_2$.

Apparently here exist surplus products y_2 - R_2 . So if they are demanded by capitalists, a strictly positive price vector can prevail, as

shown by vector **p** in Figure 2. If we draw **p** starting from each **y** and **R**, we can accordingly draw lines l_y and l_R , the normal vector of which will be **p**. The line l_y lies on the northeast of l_R , which shows there exists positive profit under **p**.

But the minimum labor necessary to produce \mathbf{R} is unity, which is as same as that of \mathbf{y} . This means there is no exploitation defined with minimized labor. Thus there can be positive profit without exploitation.

In Matsuo (1994), I proposed a new exploitation concept to solve this problem, which introduces the utility of workers. Drawing a worker's indifference curve passing through \mathbf{R} of Figure 2, we can produce \mathbf{z} as net products, which will give the same utility as \mathbf{R} to the worker, minimizing labor input to less than unity. We can call this situation "exploitation" because, just in order to maintain their welfare, workers could work less than they actually work. Then I could demonstrate that there must be no

positive profit without positive exploitation in this sense. The general proof of the equivalence between the positive profit and the positive exploitation in this sense was carried out in Matsuo (1997). The essence of it is shown in Appendix.

In Matsuo (2004), I used this exploitation concept to criticize the argument of GCET. The GCET (Bowles-Gintis(1981), Roemer(1982)) claims that we can define "value" as direct-indirect input of any commodity, say "banana", and show that the positive profit is identical to the situation wherein the banana value of the inputs to produce one unit banana is less than one unit. Allegedly, this is a criticism against the Marxian economics because this situation might be called "bananas are exploited" if Marxian says "workers are exploited", which would make it clear that Marxian economics gives a special status to only labor groundlessly and imposes an over-implication to a mathematical form.

In Matsuo (2004), I replied that, bestowed labor value system has its dual system, wherein the final input is labor and the net outputs are disposable goods for humans. Just like this, bestowed banana value system also has its dual system, wherein the final input is banana and the net output bundle must contain inputs to produce banana. The consumption goods for workers are not contained in this net output bundle. For the economy represented by the system, the net output is the object and the final input is the means to be economized. Therefore using bestowed labor value system means standing at the worker's point of view. Just as this, using bestowed banana value system means standing at the banana's point of view. MFT criticizer by GCET claims that saying "workers are exploited" is nonsense just as saying "bananas are exploited" is nonsense. But I replied that saying "bananas are exploited" is nonsense is to stand at the banana's point of view. What is nonsense is to stand at the banana's point of view, that is, to use bestowed banana value system. Contrary to this, standing at worker's point of view is not nonsense, thus using bestowed labor value system is not nonsense, thus saying "worker's are exploited" is not nonsense.

In this context, I applied the above exploitation concept defined with equal utility to GCET. Then, if we use bestowed banana value, we must solve minimization problem of banana input and use *banana's utility function*, for the conditions to be compatible. This clearly shows that this argument makes little sense.

Yoshihara (2005) criticized this exploitation concept at two points.

First. As **p** is the price vector, line $l_{\rm R}$ is the budget constraint line of the worker.

Then if **R** is chosen as the optimum, the line is the tangent to the indifference curve. Therefore z is at the outside of the budget constraint. This means I evaluate the exploitation by the goods that the worker cannot actually buy.

Second. By this evaluation of the exploitation, different workers with different preferences would suffer different amount of the exploitation even if the economic condition is the same.

Besides, Yoshihara (2005) pointed out that if we apply some exhaustible resources for the bestowed value, the utility function here represents some evaluation function of human concerned with the economization of the resources. I agree this. But if so, saying "water is exploited" is not nonsense. Thus saying "workers are exploited" is not nonsense either.

4. My Refutation in Matsuo (2006)

In Matsuo (2006), I refute the Yoshihara's (2005) criticism to my new exploitation concept. On the first point, I feel no problem about evaluating the exploitation by the goods that the worker cannot actually buy. Because the bestowed labor value system is the normative evaluation system at the standpoint of the working class. Morishima's labor minimization criterion to evaluate the bestowed labor value also uses the production process that the capitalists do not actually use. Exploitation means that if we ruled the whole production processes, we could work less than we actually work. This is not the description of the real economy.

On the second point, I feel also no problem about the difference of the amount of the exploitation among the workers of different preferences. The quantitative comparison of the exploitation between individual workers is not the important matter. The important matter is whether there exists exploitation or not. The condition of the existence of the exploitation in my proof requires holding under all utility functions in the plausible utility function set. Therefore whether there exists exploitation or not is the objective matter among the working class.

I think the impression that the amount of the labor value evaluation must be objectively determined is the vestige of the labor value theory of prices. But we now know the price system of the capitalist economy and the bestowed labor value system is the different thing. We do not need any labor value theory of prices to prove the existence of the exploitation of labor. Income distribution measured by prices is relatively different matter from the quantitative condition of the surplus labor. Marx wrote the *Capital* under the era of commonsense of the labor value theory of prices, to insist that the price form becomes alienated from the essence of the labor value determination. To separate the labor value theory from the price theory is his intention.

Yoshihara's (2005) criticism against FMT by his proposition of the existence of unexploited individual can be refuted also from the viewpoint that the bestowed labor value evaluation is the normative system for the working class. See Figure 1. Indeed, 1.5 units of goods-1 that the type-1 worker gets from a unit labor supply would need 1.5 units of labor to produce if there were only type-1 worker in this economy. But if so, positive profit could not exist. Actually positive profit can exist because both types of workers exist and the real wage bundle can be averaged to produce effectively. Then, also in the normative situation where each worker evaluates the bestowed labor of the real wage bundle, the fact that there are two types of worker must be considered. If type-1 worker works 0.75 units labor and type-2 worker works 0.75 units labor, then 1.5 units of goods-1 and 1.5 units of goods-2 are produced. Type-1 worker can get 1.5 units of goods-2 morker set worker on the types of workers can get the same amount of goods as they actually get from one unit of labor supply each, working only 0.75 units of labor each. Thus we can say that the all workers are exploited also in this situation.

I can generally prove the equivalence between the positive profit and the positive exploitation of labor under the condition of the different consumption bundle among individual workers, if we define the positive exploitation as the situation that the commodity bundle in which all workers can share their same consumption bundle they actually get from one unit of labor supply, can be produced if every worker works less than unity.

Professor Yoshihara's comment on this solution is that this type of labor value would change as the composition of the worker's types of the preferences change, even if the production techniques and distribution of goods were fixed. He says this concept is different from the traditional concept of Marxian labor value. I agree this.

5. Remaining Difference

My interpretation of the concept of the exploitation is that it says workers are forced to work to produce net products that they cannot dispose by their free will. Professor Yoshihara says that such kind of situation can be expressed just by pointing out the existence of the surplus products and the necessity of the labor input, without the expression of the exploitation. But I think such expression is necessary to express the situation as the load of every individual worker.

My achievement through this controversy is to purify the exploitation concept to the normative concept. That is, the exploitation concept indicates the criticism against the actual economy comparing to the ideal criterion wherein each worker can act with self-determination and agreement all the time. Then ultimately, Professor Yoshihara's criticism comes down to the indication that such labor exploitation concept would not be appropriate for the contemporary normative concept. He says this would be a kind of libertarian's self-ownership principle, from that viewpoint redistribution to those who cannot work could not be justified. I think the activities for those who cannot work are not the exploitation if they are performed by free agreement. Professor Yoshihara regards it as too optimistic but I think if someone felt to be forced without agreement to work for those who cannot work, we should say the person is "exploited" straightforwardly, for we should realize it as a regrettable matter, and should continue efforts to persuade the need for the redistribution or to improve the situation.

Instead of the labor exploitation concept, Professor Yoshihara recommends to apply the Roemer's egalitarian concept of "exploitation." It is also a kind of normative concept, which indicates the criticism against the actual economy comparing to the ideal criterion. But here, the criterion is the situation wherein all alienable assets of society are equally distributed to all persons.

In principle, there is left in my theory a question that why the existence of the redistribution without anonymous agreement among working class can be justified, even though it is evaluated as regrettable situation. I think the bestowed labor value concept as a normative criterion must desire all person's one-hour activity as to be equally free. But to consider what the "equality of free" is, is concerned with a matter of egalitarian discussion. Contrary to this, Professor Yoshihara does not design the coup d'état to put his ideal distribution into force. He wants to create an agreeable social welfare function to evaluate the distributive justice. But to consider what the "agreeable" is, is concerned with a matter of libertarian discussion. Then we must elaborate the argument what the "free" is, what the "agree" is or what the "equality" is, and so on.

Appendix

This proof shown below is a part of what is presented in Matsuo (1997) in Japanese language. The demonstration is improved from then.

Symbols I use here are as follows.

Parameters

B_0: m×n output matrix. A≥0: m×n input matrix. R≥0: m×1 real wage basket for unit labor. $\tau \ge 0$: 1×n direct labor input coefficient vector.

Variables

p: $1 \times m$ price vector. **x**: $n \times 1$ activity vector. *T*: labor time (scalar).

And we take these assumptions.

Assumption 1: A, B and τ satisfy the axiom of impossibility of the land of Cockaigne and the condition for possibility of net production. That is;

For any $y \ge 0$, there exist $x \ge 0$, $(B-A)x \ge y$, and for this x, $\tau x > 0$.

Assumption 2: Worker's utility function u is an element of the function set U as follows; $U=\{u: \mathbb{R}^{m}_{+} \times \mathbb{R}^{1}_{+} (\ni (\mathbf{y}, T)) \rightarrow \mathbb{R}^{1} |$ (i) $\mathbf{y}^{1} \ge \mathbf{y}^{2} \Rightarrow u(\mathbf{y}^{1}, T) > u(\mathbf{y}^{2}, T)$, (ii) $T^{1} > T^{2} \Rightarrow u(\mathbf{y}, T^{1}) < u(\mathbf{y}, T^{2})$, (iii) u is continuous. }

Note that this assumption does not need to introduce notions of differentiability nor quasi-concavity. It just requires continuous and strictly increasing function for goods distribution and decreasing function for working hours. Therefore, each element of U is the function of individual worker but U covers almost all possible ways of evaluation by workers.

Now I will introduce some definitions.

Definition 1: The Weak Profit Warranty Condition is defined as follows;

$\neg \exists p > 0, p(B-A-R\tau) \leq 0$

or "There is no positive price vector, under which no process can yield positive profit". This demands existence of positive profits at some processes, under any positive price vector.

Definition 2: The Weak Surplus Condition is defined as follows;

 $\exists x \geq 0, (B - A - R\tau)x \geq 0$

or "There is an semi-positive activity vector which yields at least one type of surplus product."

Definition 3: For given $u \in U$, goods basket $\mathbf{y}^0 \ge \mathbf{0}$'s labor value defined with Minimized Labor for Equal Utility (MLEU), $v = v(\mathbf{y}^0, T; u)$, is as follows;

$$v=\min_{\mathbf{y},\mathbf{x}} \mathbf{\tau}\mathbf{x} \quad \text{s.t.} \ (\mathbf{B}-\mathbf{A})\mathbf{x} \ge \mathbf{y}, \mathbf{x} \ge \mathbf{0}, \ u(\mathbf{y},T) \ge u(\mathbf{y}^0,T)$$
(*)

Using this definition of the exploitation, I solve the Petri(1980)'s problem to rescue the FMT. That is, I prove "The Weak Profit Warranty Condition \Rightarrow The Weak Surplus Condition", "The Weak Surplus Condition $\Rightarrow \forall u \in U$, The existence of the exploitation defined with MLEU", and " $\forall u \in U$, The existence of the exploitation defined with MLEU \Rightarrow The Weak Profit Warranty Condition".

Before the first proof, I must show a lemma.

Lemma (Nikaido(1961), pp. 157-158):

Let $\mathbf{D}=[d_{ij}]$ be m×n matrix. $\mathbf{x}=(x_1, \dots, x_n)'$, $\mathbf{p}=(p_1, \dots, p_m)$. If inequality $\mathbf{D}\mathbf{x}\geq \mathbf{0}$, $\mathbf{x}\geq \mathbf{0}$ does not have a solution \mathbf{x} , then there exist solutions \mathbf{p} for inequality $\mathbf{p}\mathbf{D}\leq \mathbf{0}$, $\mathbf{p}>\mathbf{0}$.

Proposition 1: The Weak Profit Warranty Condition \Rightarrow The Weak Surplus Condition Proof:

Substituting **D** of the *Lemma* as **B**-**A**-**R** τ , then

 $\neg \exists p > 0, p(B-A-R\tau) \leq 0 \Rightarrow \exists x \geq 0, (B-A-R\tau)x \geq 0$

The right hand condition is equivalent to the Weak Surplus Condition because if x=0, $(B-A-R\tau)x$ would be zero.

(q. e. d.)

Proposition 2: The Weak Surplus Condition $\Rightarrow \forall u \in U, 1 - v(\mathbf{R}, 1; u) > 0$

Proof:

From $\exists x \ge 0$, (**B**-**A**-**R** τ)**x** ≥ 0 , adjusting the scale of **x** to be $\tau x^0 \equiv 1$, we get,

 $\exists \mathbf{x}^{0} \geq \mathbf{0}, (\mathbf{B} \cdot \mathbf{A}) \mathbf{x}^{0} \geq \mathbf{R}$

From (i) of Assumption 2,

 $u((\mathbf{B}-\mathbf{A})\mathbf{x}^{0}, 1)>u(\mathbf{R}, 1)$

From (iii) of Assumption 2,

 $\exists \alpha \in (0, 1), u((\mathbf{B} \cdot \mathbf{A}) \alpha \mathbf{x}^0, 1) > u(\mathbf{R}, 1)$

Then setting $\mathbf{y}^0 = \mathbf{R}$, $\mathbf{x} = \boldsymbol{\tau} \mathbf{x}^0$, $\mathbf{y} = (\mathbf{B} \cdot \mathbf{A}) \alpha \mathbf{x}^0$, this satisfies the constraints (*) of the *Definition 3*. Therefore, $v \leq \boldsymbol{\tau} \alpha \mathbf{x}^0$. From $\boldsymbol{\tau} \mathbf{x}^0 \equiv 1$, $\boldsymbol{\tau} \alpha \mathbf{x}^0 < 1$. \therefore 1-v > 0 (q. e. d.)

Proposition 3: $\forall u \in U$, 1- $v(\mathbf{R}, 1; u) > 0 \Rightarrow$ The Weak Profit Warranty Condition *Proof*:

Use *reductio ad absurdum*. Multiplying any $x \ge 0$ to the negation of the Weak Profit Warranty Condition, we get,

 $\exists \mathbf{p} > \mathbf{0}, \mathbf{p}(\mathbf{B} - \mathbf{A} - \mathbf{R} \tau) \mathbf{x} \leq 0$, for any $\mathbf{x} \geq \mathbf{0}$

That is, under all semi-positive " \mathbf{x} "s, there exists at least one common positive price vector that makes the total profit non-positive.

Then, adjusting the scale of **x** to be $\tau \mathbf{x}^0 \equiv 1$ for every **x**, we get,

 $\mathbf{p}(\mathbf{B}-\mathbf{A})\mathbf{x}^{0} \leq \mathbf{p}\mathbf{R}$

Therefore, for every $\mathbf{x}^0 \ge \mathbf{0}$, each, there exists $\mu \ge 1$, $\mathbf{p}(\mathbf{B}-\mathbf{A}) \ \mu \ \mathbf{x}^0 = \mathbf{p}\mathbf{R}$

Now, compose the utility function as $u(\mathbf{y}, T) \equiv \mathbf{p}\mathbf{y} - T$, then $u \in U$.

Then, we can evaluate $v(\mathbf{R}, 1)$ for this utility function. As constraint (*) of the *Definition 3* is $\mathbf{p}(\mathbf{B}-\mathbf{A})\mathbf{x} \ge \mathbf{p}\mathbf{R}$, there exists $\mu \ge 1$, $\mathbf{p}(\mathbf{B}-\mathbf{A})\mathbf{x} \ge \mathbf{p}(\mathbf{B}-\mathbf{A}) \mu \mathbf{x}^0$, for every $\mathbf{x} \ge \mathbf{0}$ of the constraint. This also stands for \mathbf{x}_* , solution of this labor minimization problem and its scalar-fold \mathbf{x}_*^0 . At this solution, the constraint above holds as an equation. And price vector is not zero vector.

Therefore, $v(\mathbf{R}, 1) \equiv \mathbf{\tau} \mathbf{x}_* = \mathbf{\tau} \ \mu \ \mathbf{x}_*^0 = \mu \ge 1$ (q. e. d.)

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